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ESTIMATES OF THE NORMAL VELOCITIES OF PROPAGATION OF LAMINAR AND VERY SMALL-SCALED TURBULENT FLAMES

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On the most general assumptions (taking account of the Lewis–Semenov number, thermal expansion, variability of thermophysical parameters, etc.), analytical estimates are obtained for the normal velocities of combustion of laminar and turbulent flames. In the case of an Arrhenius dependence of the reaction velocity on the temperature, the combustion velocity is represented by an asymptotic series with respect to the Frank-Kamenetskii dimensionless temperature; for turbulent flames, with respect to a parameter of the relative scale of turbulence. The final results over a wide range of change of parameters are compared with a numerical calculation on a computer of the exact equations and with the relations obtained by the method of combined asymptotic expansions.

1. Mathematical Formulation of the Problem. Laminar Flame

When the temperature dependence of the rate of the volume heat release is determined by the Arrhenius law

$$\Phi = (\rho(T))^n v^n z(T) \exp(-E/RT), \tag{1.1}$$

the thermal diffusion mechanism of propagation of a one-dimensional steady flame is described [1] by the system of equations

$$\begin{gathered} dp/du = v^n k(u) f(u)/p - \omega; \\ (1/L) dv/du = 1 - \omega(v-u)/p,\ 0 < u < 1 \end{gathered} \tag{1.2}$$

and with the boundary conditions

$$u = 0,\ p = 0,\ v = 0; \tag{1.3}$$

$$u = 1,\ p = 0; \tag{1.4}$$

$$f(u) = \begin{cases} \exp(-\theta_0 u/(1-\sigma u)), & 0 \leqslant u \leqslant \varepsilon \\ 0, & \varepsilon < u \leqslant 1. \end{cases} \tag{1.5}$$

The "cutoff" equation (1.5) of the heat release (ε is the "cutoff" parameter) ensures the existence of an eigenvalue ω_0 of the problem (1.1)-(1.4), which is unique when $1 \leq \mathrm{Le} < \infty$ [1]. The question of uniqueness when $\mathrm{Le} < 1$ still does not have a solution.

The relations between the dimensionless and dimensional quantities are

$$u = (T_+ - T)/(T_+ - T_-);\ p = -(\lambda/\lambda_+) du/d\xi;\ \xi = x/x_+,\ k(u) = (\lambda/\lambda_+)(\rho/\rho_+)^n z/z_+;$$

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$$\mathrm{Le} = \lambda / c\rho D;\ \beta = RT_+/E;\ x_+ = (a_+\tau_+)^{1/2};\ a_+ = \lambda_+/c\rho_+;$$

$$\tau_+ = z_+^{-1}\rho_+^{1-n}\exp(1/\beta);\quad \omega = w_+(\tau_+/a_+)^{1/2};$$

$$\theta_0 = (E/RT_+^2)(T_+ - T_-);\quad \sigma = \beta\theta_0 = 1 - T_-/T_+,$$

where v is the concentration; T is the temperature; w_+ is the normal velocity of flame propagation, relative to the reaction products; c = const is the specific heat at constant pressure; $\rho = \rho(T)$ is the density; $D = D(T)$ is the effective coefficient of diffusion; n is the reaction order; x_+ is the spatial scale; z(T) is the frequency factor; Le is the Lewis–Semenov number; τ_+ is the characteristic time of chemical reaction; E is the energy of activation; R is the gas constant. We denote the parameters referring to the initial mixture and to the final reaction products by the subscripts minus and plus, respectively. A similar indexing is used later for denoting the upper and lower bounds of functions and velocities of combustion.

In the special case of power functions $\lambda \sim T^{m_1}$, $\rho \sim T^{-n}$, $z \sim T^{m_2}$, $0 \le m_1 \leqslant 1$, $0 \le n \le 3$, $0 \le m_2 \leqslant 1$, we have $k(u) = (1 - \sigma m)^m$, $m = m_1 - n + m_2$.

Further consideration is valid also in the case when the temperature dependence f(u) differs from the Arrhenius dependence, but satisfies the conditions for the existence and uniqueness of the eigenvalue ω_0.

The case of the dependence of the solutions of system (1.2) with the conditions (1.3) on ω is denoted by the following equations:

$$p(u) = \bar{p}(\omega, u);\ v(u) = \bar{v}(\omega, u).$$

Obviously,

$$dp/du = \partial\bar{p}/\partial u;\ dv/du = \partial\bar{v}/\partial u. \tag{1.6}$$

The boundary condition (1.4), taking account of Eq. (1.5), is equivalent to the condition

$$p(\varepsilon) - \omega(1 - \varepsilon) = 0. \tag{1.7}$$

Therefore, instead of Eq. (1.4), Eq. (1.7) can be used, and the solutions of system (1.2) can be considered only in the region $0 < u < \varepsilon$.

2. Estimates of Combustion Velocities. Laminar Flame

When Le = 1, system (1.2) reduces to a single equation:

$$dp/du = \varphi(u)/p - \omega,\ \varphi(u) = u^n k(u) f(u). \tag{2.1}$$

We shall assume the "cutoff" parameter to be variable and we shall denote it by t, $0 < t < \varepsilon$. The eigenvalue $\omega_0 = \omega_0(t)$ will be satisfied according to Eq. (1.7) by the equation

$$\bar{p}(\omega_0, t) - \omega_0(1 - t) = 0. \tag{2.2}$$

Differentiating this equation with respect to t, and taking into account that according to Eq. (1.6)

$$\partial\bar{p}/\partial t + \omega_0 = \lim_{u\to t}(dp/du + \omega_0) = \varphi(t)/\bar{p}(\omega_0, t) = \varphi(t)/\omega_0(1 - t),$$

and denoting $q(t) = \bar{q}(\omega, t) = \partial\bar{p}(\omega, t)/\partial\omega$, we obtain the differential equation for ω_0:

$$d\omega_0/dt = \varphi(t)/\{\omega_0(1-t)[1 - t - \bar{q}(\omega_0, t)]\} \tag{2.3}$$

with the condition $\omega_0(0) = 0$, which follows from Eq. (2.2)

According to the theorem of estimates [2], with increase of ω the solution of Eq. (2.1) with the condition $p(0) = 0$ is reduced; therefore, $q < 0$. Differentiating Eq. (2.1) with respect to ω, we obtain

$$dq/du = -(\varphi(u)/p^2)q - 1.$$

When u = 0, q = 0, and therefore

$$q(t) = -\int_0^t \frac{\varphi(u)}{p^2} q\,du - t > -t.$$

After substitution in Eq. (2.3) of the upper ($q_+ = 0$) and lower ($q_- = -t$) functions and after subsequent integration, we find the upper and lower estimate of ω_0:

$$\omega_+^2 = 2\int_0^\varepsilon \frac{\varphi(u)}{(1-u)^2}\,du;\quad \omega_-^2 = 2\int_0^\varepsilon \frac{\varphi(u)}{1-u}\,du. \tag{2.4}$$

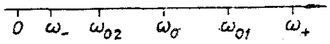

Fig. 1

Similar relations are obtained in [3] by a more complex method. The approximate formula of Zel'dovich is well-known [4]:

$$\omega_\infty^2 = 2\int_0^\varepsilon \varphi(u)\,du. \tag{2.5}$$

Formula (2.5) gives a value for ω_∞ which is less than the lower estimate of Eq. (2.4).

For the two equations of (1.2), similarly to the foregoing, we have

$$d\omega_0/dt = [\overline{v}^n(\omega_0, t)k(t)f(t)]/\{\omega_0(1-t)[1-t-\overline{q}(\omega_0, t)]\}. \tag{2.6}$$

Equation (2.6) can be used for finding an estimate of ω_0, if the estimates of v and q are known. A simpler route consists in the use of formulas (2.4). We shall suppose that an upper function $v_+(u)$ has been found, independent of ω, such that $v < v_+$. The eigenvalue ω_{01} of the problem

$$dp_1/du = v_+^n k(u)f(u)/p_1 - \omega, \quad p_1(0) = p_1(1) = 0$$

will be greater than ω_0. Actually, in considering the solution of this equation with the condition $p_1(0) = 0$, we have

$$p_1(1) = \overline{p}_1(\omega_0, \varepsilon) - \omega_0(1-\varepsilon) > \overline{p}(\omega_0, \varepsilon) - \omega_0(1-\varepsilon) = 0,$$

and, in order that $p_1(1)$ vanishes, it is necessary to increase ω. It can be seen in Fig. 1 that the upper estimate for ω_{01}, which can be obtained by means of Eq. (2.4), will be the upper estimate also for ω_0.

In a similar way, it will be shown that if $\overline{v}(\omega_0, u) > v_-(u)$, the lower estimate ω_{02} for the eigenvalue of the problem

$$dp_2/du = v_-^n(u)k(u)f(u)/p_2 - \omega, \quad p_2(0) = p_2(1) = 0$$

is the lower estimate also for ω_0 (see Fig. 1).

A. The Case $1 \le \mathrm{Le} < \infty$. For simplicity we put Le = const. We shall show that

$$v_-(u) = u < \overline{v}(\omega, u) < 1 - (1-u)^{\mathrm{Le}} = v_+(u). \tag{2.7}$$

The expansion of the solution of system (1.2) and (1.3) in series of powers of u in the vicinity of u = 0 has the form

$$p = p_0'u + p_0''u^2/2 + \ldots; \quad v = v_0'u + v_0''u^2/2 + \ldots.$$

The expansion coefficients are equal to

$$p_0' = \frac{-\omega + \sqrt{\omega^2 + 4k_0\mathrm{Le}^{-1}}}{2\mathrm{Le}^{-1}}; \quad v_0' = \frac{\omega + p_0'}{\omega + \mathrm{Le}^{-1}p_0'}, \quad k_0 = k(0), \quad n = 1;$$

$$p_0' = 0, \quad p_0'' = 2k_0/\omega; \quad v_0' = 1, \quad v_0'' = (2k_0/\omega^2)(1 - \mathrm{Le}^{-1}), \; n = 2$$

(solutions are considered which are positive in the vicinity of u = 0). It can be seen that v > u in the vicinity of u = 0 for any values of ω. If we suppose that v > u when $0 < u < u_0$ and $v(u_0) = u_0$, then at this point we should have $dv(u_0)/du \le 1$; in reality [which follows from the second equation of (1.2)], $dv(u_0)/du = \mathrm{Le} > 1$. The contradiction obtained proves the left-hand side of inequality (2.7).

It follows from the first equation of (1.2) that

$$(d/du)(p + \omega u) = k(u)v^n f(u)/p - \omega,$$

whence follows the inequality $\overline{p}(\omega_0, u) + \omega_0 u < \overline{p}(\omega_0, \varepsilon) + \omega_0\varepsilon$. Taking account of Eq. (1.7), we obtain $\overline{p}(\omega_0, u) < \omega_0(1-u)$. Substituting the quantity $\omega_0(1-u)$ in place of p in the second equation of (1.2) and solving it, we arrive at the upper estimate v_+; inequality (2.7) is proved.

In accordance with what has been stated, the estimates assume the form

$$\omega_+^2 = 2\int_0^\varepsilon \frac{[1-(1-u)^{\mathrm{Le}}]^n k(u)f(u)}{(1-u)^2}\,du; \tag{2.8}$$

$$\omega_-^2 = 2\int_0^{\varepsilon} \frac{u^n k(u) f(u)}{1-u}\, du. \quad (2.9)$$

If Le ≠ const, then

$$v_+(u) = 1 - \exp\left(\int_0^u \frac{\mathrm{Le}(u)\, du}{1-u}\right)$$

and estimate (2.8) changes in a corresponding manner.

B. The Case $0 < \mathrm{Le} < 1$. The inequality $v_- = 1 - (1-u)^{\mathrm{Le}} < v < u = v_+$ is proved by similar considerations for Le = const. In all, we have

$$\omega_+^2 = 2\int_0^{\varepsilon} \frac{u^n k(u) f(u)}{(1-u)^2}\, du; \quad (2.10)$$

$$\omega_-^2 = 2\int_0^{\varepsilon} \frac{[1-(1-u)^{\mathrm{Le}}]^n k(u) f(u)}{1-u}\, du. \quad (2.11)$$

When Le ≠ const, v_- and the lower estimate of Eq. (2.11) are changed.

If certain eigenvalues exist, then they are all included between ω_+ and ω_-, determined by formulas (2.10) and (2.11). The uniqueness of ω_0 in the case $\mathrm{Le} \geq 1$ follows from the monotonicity of the function $\bar{p}(\omega, 1) = \bar{p}(\omega, \varepsilon) - \omega(1-\varepsilon)$ [it is shown in [1] that with increase of ω the function $\bar{p}(\omega, 1)$ decreases ($\bar{q}(\omega, 1) < 0$)]. In the case $\mathrm{Le} < 1$, it is not possible to show analytically the negativity of $\bar{q}(\omega, 1)$. By means of a numerical experiment for the parameters $\sigma \leq \theta_0 \leq 14$, $0.1 \leq \mathrm{Le} < 1$ and different values of σ, we shall show that $\bar{q}(\omega, 1) < 0$ and, consequently, ω_0 is unique.

C. The Case $\mathrm{Le} = \infty$. The upper estimate is obtained from Eq. (2.8), determined by the transition ($\mathrm{Le} \to \infty$)

$$\omega_+^2 = 2\int_0^{\varepsilon} \frac{k(u) f(u)}{(1-u)^3}\, du. \quad (2.12)$$

The lower estimate (2.9) is unchanged. A better lower estimate than Eq. (2.9) can be obtained if we start not from system (1.2), but from one equation to which this system is reduced for $\mathrm{Le} = \infty$:

$$dp/du = (u + p/\omega)^n k(u) f(u)/p - \omega.$$

The equation for ω_0 assumes the form

$$d\omega_0^2/dt = [2k(t) f(t)] / \{(1-t)[1 - t - \bar{q}(\omega_0, t)]\}. \quad (2.13)$$

When $n = 1$, we have

$$dq/du = -(uk(u) f(u)/p^2) q - (k(u) f(u)/\omega^2) - 1.$$

Taking into account that $q < 0$, we obtain

$$q > -t - \frac{1}{\omega^2}\int_0^t k(u) f(u)\, du > -t - \frac{1}{\omega^2}\int_0^t \frac{k(u) f(u)}{1-u}\, du = q_-.$$

Substituting q_- instead of q in Eq. (2.13), we find

$$\omega_-^2 = \int_0^{\varepsilon} k(u) f(u) (1-u)^{-1} du. \quad (2.14)$$

In the case $n = 2$, since $(u + p/\omega)^2 > u^2 + (2u/\omega p)$, the lower estimate for the eigenvalue of the problem

$$dp/du = [(u^2 + 2up/\omega) k(u) f(u)/p] - \omega, \; p(0) = p(1) = 0 \quad (2.15)$$

will be the lower estimate also for ω_0. The equation for ω_0 of problem (2.15) has the form

$$\frac{d\omega_0^2}{dt} = \frac{2t(2-t) k(t) f(t)}{(1-t)[1 - t - \bar{q}(\omega_0, t)]}. \quad (2.16)$$

Differentiating Eq. (2.15) we respect to ω

$$dq/du = -(u^2 f(u)k(u)/p^2)q - (2uf(u)k(u)/\omega^2) - 1$$

and taking into account that $q<0$ and $2<(2-t)/(1-t)$, we shall have

$$q_- = -t - \frac{1}{\omega^2}\int_0^t t\frac{2-t}{1-t}f(t)k(t)\,dt. \tag{2.17}$$

Substituting Eq. (2.17) in place of q in Eq. (2.16), we obtain

$$\omega_-^2 = \int_0^\varepsilon \frac{2-t}{1-t}tk(t)f(t)\,dt. \tag{2.18}$$

For n = 1, it is found in [5] that $\omega_-^2 = \int_0^\varepsilon f(u)k(u)du$ and for n = 2

$$\omega_-^2 = 4\int_0^\varepsilon uf(u)k(u)\,du.$$

The estimate (2.14) is more accurate than in [5].

3. Approximate Relations. Laminar Flame.

The integrals occurring in the estimate for the case of a strong dependence of f(u) on temperature ($|d\ln f(u)/du| \gg 1$) will be derived approximately. The approximation consists in the following: all functions which do not depend too strongly on temperature $(1-u)^{-1}$, $(1-u)^{-2}$, $k(u) = (1-\sigma u)^m$, and $[1-(1-u)^{\mathrm{Le}}]^n$ are expanded in a power series in the vicinity of the maximum temperature of combustion (u = 0). The function $f(u) = \exp[-\theta_0 u/(1-\sigma u)]$ also is represented by a series $f(u) = (1-\theta_0\sigma u^2 + \ldots)e^{-\theta_0 u}$.

As a result of term-by-term multiplication of the series, a series is obtained whose coefficients are proportional to integrals of the form

$$\int_0^\varepsilon u^{n+s}e^{-\theta_0 u}du. \tag{3.1}$$

expressed in terms of an incomplete Γ-function.

The final result for the velocity ω should be independent of the "cutoff" parameter ε; in the contrary case, the initial formulation of the problem (1.2)-(1.4) becomes physically incorrect. The independence of ω on ε is equivalent to substituting the upper integration limit ε in Eq. (3.1) by infinity, or, what amounts to the same, by neglecting terms of order $(\theta_0^n e^{-\theta_0\varepsilon})$ in comparison with unity. Therefore, instead of Eq. (3.1) we have

$$\int_0^\infty u^{n+s}e^{-\theta_0 u}du = \frac{\Gamma(n+s+1)}{\theta_0^{n+s+1}}.$$

Let us confine ourselves to the case k(u) = 1.

We write certain asymptotic expansions obtained in this way with an accuracy of $(1/\theta_0)$:

$$\omega_- = \omega_\infty[1 + (n+1)/2\theta_0(1-(n+2)\sigma - n(\mathrm{Le}-1)/2)], \tag{3.2}$$

where

$$\omega_\infty = \sqrt{\frac{2\Gamma(n+1)\,\mathrm{Le}^n}{\theta_0^{n+1}}} \tag{3.3}$$

is the asymptotic formula of Ya. B. Zel'dovich–L. D. Landau. Formula (3.2) is valid for $0 < \mathrm{Le} < 1$.

Over the range of variation of $1 \le \mathrm{Le} < \infty$, the estimate above Eq. (2.8) has the form

$$\omega_+ = \omega_\infty[1 + (n+1)/2\theta_0\cdot(2-(n+2)\sigma - (n/2)(\mathrm{Le}-1))]. \tag{3.4}$$

It can be seen that the difference between Eq. (3.2) and (3.4) is not very significant. Therefore, the arithmetic mean will be used as the interpolation formula, consisting of ω_- and ω_+. The result obtained in this way with some approximation can be extended to the entire range of variation of $0 < \mathrm{Le} < \infty$. The basis for this extension is also that the estimate (2.8) is the upper estimate for $\omega_{02} < \omega_0$ (see Fig. 1) and in the case Le >1 the estimate (2.11) is the lower estimate for $\omega_{01} > \omega_0$:

$$<\omega> = (\omega_+ + \omega_-)/2 = \omega_\infty \{1 + [(n+1)/2\theta_0](3/2 - (n+2)\sigma - (n/2)(\mathrm{Le}-1))\}. \tag{3.5}$$

When Le is finite and $\theta_0 \gg 1$, the upper (3.4) and lower (3.2) estimates coincide, so that formula (3.3) also has a an asymptotic meaning when Le ≠ 1. In a number of cases, it will give a good result also for values of θ_0 which are not too large (Table 1) because of compensation in the signs of the expansion coefficients of order (θ_0^{-1}). In the case Le → ∞, we find from Eq. (2.14) and (2.18)

$$\omega_- = (1/\sqrt{\theta_0})[1 + (0{,}5 - \sigma)/\theta_0],\ n = 1; \tag{3.6}$$

$$\omega_- = (\sqrt{2}/\theta_0)[1 + (0{,}5 - 3\sigma)/\theta_0],\ n = 2. \tag{3.7}$$

Comparison of formulas (3.2) and (3.4)-(3.7) with the results obtained by the method of combined asymptotic expansions (CAE) [6, 7], given in Table 2, where a, b, and c are the expansion coefficients of order (1/0), is represented in the form $\theta_0^{-1}[a - b\sigma - c(\mathrm{Le}-1)]$. For Le = ∞, $\omega_\infty(n=1) = 1/\sqrt{\theta_0}$ and $\omega_\infty(n=2) = \sqrt{2}/\theta_0$. Comparison with the numerical calculation of the boundary-value problem on a computer is given in Tables 1 and 3. Here ω_0 (computer) is the numerical calculation; $\Delta_\infty = \omega_\infty/\omega_0$, $\Delta(\mathrm{CAE}) = \omega[6,7]/\omega_0$, $\Delta_- = \omega_-/\omega_0$, and $<\Delta> = <\omega>/\omega_0$ are the corresponding deviations from the exact value of the velocity.

4. Estimates of the Combustion Velocity. Very Slightly Turbulent Flame

The combustion velocity ω_0 is the eigenvalue of the problem [8, 9]

$$dp/du = \Phi(u,p)/p - \omega,\ p(0) = p(1) = 0,$$
$$2\Phi(u,p) = \begin{cases} \varphi(u+Fp) + \varphi(u-Fp), & 0 \leqslant u \leqslant \varepsilon \\ 0, & \varepsilon < u \leqslant 1, \end{cases} \tag{4.1}$$
$$\varphi(u) = u^n f(u).$$

The derivation of the equation for ω_0 is similar to the derivation of Eq. (2.3),

$$d\omega_0/dt = \Phi(t, \omega_0(1-t))/\omega_0(1-t)(1-t-q),\ \omega_0(0) = 0. \tag{4.2}$$

According to the theorem of estimates [2], $q = \partial\bar{p}(\omega,u)/\partial\omega < 0$. Assuming that q = 0, we find from Eq. (4.2) the equation for determining ω_+:

$$d\omega_1/dt = \Phi(t, \omega_1(1-t))/\omega_1(1-t)^2,\ \omega_1(0) = 0,\ \omega_1(\varepsilon) = \omega_+. \tag{4.3}$$

By means of Eq. (4.1), we obtain

$$q = -t + \int_0^t \frac{\partial}{\partial p}\left(\frac{\Phi}{p}\right) q\,du. \tag{4.4}$$

With small values of Fp in expansion in powers of Fp

$$(\partial/\partial p)(\Phi/p) = (1/p^2)\{-\varphi(u) + \varphi''(u)(Fp^2)/2 + \ldots\}$$

the principal contribution is made by the first term in the curly brackets, and there $\partial/\partial p\cdot(\Phi/p) \leqslant 0$. Using this inequality, we find from Eq. (4.4) that q > −t. The equation for determining ω_- is obtained from Eq. (4.2), if we put q = −t.

$$d\omega_2/dt = \Phi(t, \omega_2(1-t))/\omega_2(1-t),\ \omega_2(0) = 0,\ \omega_2(\varepsilon) = \omega_-. \tag{4.5}$$

Table 4 shows the results of a numerical computation ω_0 (computer), ω_+, ω_-, and $\Delta = <\omega>/\omega_0$.

In order to obtain the approximate relations in the expansion of Φ in powers of Fp (the case of small temperature pulsations), we limit ourselves to two terms:

$$\Phi \approx \varphi(u) + \varphi''(u)F^2p^2/2.$$

Substituting this approximate value of Φ in Eq. (4.3) and (4.5), we find

$$\omega_+^2 = \omega_1^2(\varepsilon) = 2\int_0^\varepsilon \frac{\varphi(u)}{(1-u)^2} \exp[-F^2\varphi'(u)]\,du; \tag{4.6}$$

$$\omega_-^2 = \omega_2^2(\varepsilon) = 2\int_0^\varepsilon \frac{\varphi(u)}{1-u} \exp[-F^2(\varphi(u) + \varphi'(u)(1-u))]\,du.$$

TABLE 1

Le^{-1}	$n=1$, $\sigma=0,6$, $\theta_0=10$					$n=1$, $\sigma=0.9$, $\theta_0=10$				
	ω_0 computer	Δ_∞	ΔCAE	Δ_-	<Δ>	ω_0 computer	Δ_∞	Δ CAP	Δ_-	<Δ>
0.2	0.234	1.35	0,75	0,97	1,04	0.221	1,43	0.67	0.90	0.97
0.6	0.167	1.08	0.96	0.96	1.02	0.155	1.17	0.94	0.94	0.99
1.0	0.136	1.04	0.99	0.96	1.01	0.127	1.11	0.97	0.92	0.98
6	0.061	0.95	0,98	0.92	0.97	0.055	1.05	1	0.93	0.96
10	0,047	0.96	1,00	0.94	0.98	0.043	1.05	1	0.91	0.98

TABLE 2

n		ω_- (3.2) $0<Le\leqslant 1$	ω_+ (3.4) $1\leqslant Le<\infty$	ω [6] $Le\sim O(1)$	ω_- (3.6) (3.7) $Le=\infty$	ω [7] $Le=\infty$
1	a	1	2	1.344	0,5	0.82
	b	3	3	3	1	1
	c	0.5	0.5	1	—	—
2	a	1.5	3	2.11	0.5	—
	b	6	6	6	3	—
	c	1.5	1.5	4.442	—	—

TABLE 3

θ_0	$Le=\infty$				$\sigma=0.9$		
	ω_+ (2.12)	ω_0 computer	ω_- (2.14)	ω_- (3,6)	ω [7]	Δ_-	ΔCAE
6	0.587	0.398	0.386	0.381	0.403	0.96	1.01
9	0,477	0.323	0.321	0,319	0.330	0.99	1.02
10	0.452	0.306	0.305	0.304	0.314	0,99	1.03
12	0.412	0.280	0,280	0,279	0,287	1	1,03

TABLE 4

θ^0	$\sigma=0.8$, $n=1$				
	F	ω_+	ω_0	ω_-	Δ
6	1	0,2306	0,2109	0,1938	1.01
	4	0.3218	0,2511	0.1959	1.03
	7	0,4755	0,3352	0.2130	1.02
14	1	0.0995	0.0962	0.0918	0.99
	4	0.1238	0,1101	0.0967	1.00
	8	0.1904	0.1568	0,1135	0.97

TABLE 5

θ_0	F	ω_0	$\langle\omega\rangle/\omega_0$
6	1	0.2109	1.00
	4	0.2511	1,42
14	1	0.0962	1,02
	4	0.1101	1,12

If we expand the exponential functions in Eq. (4.6) in series with respect to the exponent and we limit ourselves to two terms of the expansion, then

$$\omega_+^2 = 2\int_0^\varepsilon \frac{\varphi(u)}{(1-u)^2}\,du + 2F^2\int_0^\varepsilon \frac{\varphi^2(u)}{(1-u)^3}\,du; \tag{4.7}$$

$$\omega_-^2 = 2\int_0^\varepsilon \frac{\varphi(u)}{1-u}\,du - 2F^2\int_0^\varepsilon \frac{\varphi^2(u)}{1-u}\,du.$$

The final result is obtained if, in the evaluation of the integrals in Eq. (4.7), we use the same method as in the case of the laminar flame:

$$\langle\omega\rangle = \omega_\infty\,\{1 + [(n+1)/2\,\theta_0](3/2 - (n+2)\,\sigma)\} + [(3n+1)\Gamma(2n+1)/2\omega_\infty\,(2\,\theta_0)^{2n+1}]F^2. \tag{4.8}$$

Comparison of the approximate values of the combustion velocity, calculated by formula (4.8) for the same parameters as in Table 4, with exact values of ω_0 is given in Table 5.

The method of estimates, in contrast from the method of combined asymptotic expansions, has the drawback that it does not consist of the standard procedure for finding the initial "best" estimate. However, once it has been found, subsequent estimates can be refined by the standard procedures for example, by Chaplygin's method [10]. Among the advantages of the method of estimates, we should make mention of the following: first, the simplicity in finding subsequent expansions in powers of $1/\theta_0$, second, a knowledge of the range over which the exact solution is applicable; and the simplicity of generalization to a heat release function which is different from the Arrhenius function. Preliminary results of this paper as applicable to laminar flames are discussed in [11].

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NONLINEAR ANALYSIS OF THE FLOW INITIATED BY THE SUDDEN MOTION OF A WEDGE

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Certain self-similar problems involving the sudden motion of a wedge which were treated in the linear approximation in [1-3] are studied by the method of matched asymptotic expansions. The nature of the wave boundary of the perturbed region is determined. Second-approximation solutions are constructed which describe flows behind weak shock fronts propagating in a stationary gas and behind fronts of weak discontinuity lines propagating by known uniform flows. A boundary-value problem is formulated whose solution describes, in first approximation, flows in the neighborhoods of points of interaction of the fronts. The existence of similarity rules of flows in these nieghborhoods is estimated. An approximate solution of the problems is given.

§1. Let us consider the flow of a stationary ideal polytropic gas arising from the sudden motion of an infinite wedge with constant velocity W_0 in the negative Ox direction. The parameters of this self-similar problem are the Mach number of the wedge $M_0 = W_0/a_0$, the adiabatic exponent of the gas γ, and the angles α_1, α_2 between the edges of the wedge of the Ox axis. The following cases are examined; a) a wedge with arbitrary vertex angle moving at low velocity $M_0 \ll 1$; b) a thin wedge $\alpha_j = \alpha \ll 1$ moving with subsonic velocity; c) a thin wedge moving with supersonic velocity (Fig. 1a-b, c, respectively). In the latter case the condition $\alpha M_0 \ll 1$ must be satisfied.

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